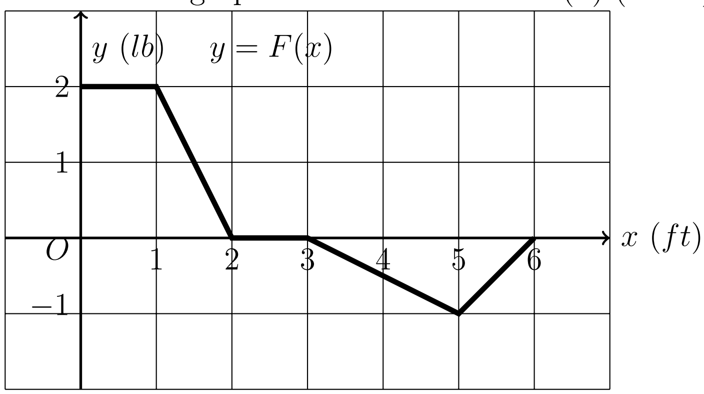
Practice Midterm1 (Take-home quiz 6, due on Tue, Oct 11)

Q1 Below is the graph of a force function $F(x)$ (in lbs).



(a) How much work is done by the force in moving an object from $x = 0$ to $x = 3$?

(b) How much work is done by the force in moving an object from $x = 0$ to $x = 5$?

Q2 Find the derivatives of the following functions

(a) $f(x) = [\tan^{-1} x]^{\ln(2x)}$, find $f'(x)$

(b) $f(x) = \pi^{\sin^{-1}(\sqrt{x})}$, find $f'(x)$

(c) $f(x) = x^2 + 3\sin x + 1$, find $(f^{-1})'(1)$

Q3 Determine whether the following limits exist or not. Find the limit if it exists.

(a)

$$\lim_{\theta \to \pi/2} \frac{1 - \sin\theta}{1 + \cos(2\theta)}$$

(b)

$$\lim_{x \to +\infty} \sqrt{x} \cdot e^{-x/2}$$

(c)

$$\lim_{t \to 0^+} t \cdot \tan^{-1}(1/t)$$

Q4 Evaluate the following integrals

(a)

$$\int \sin(3t) \cdot (2t + 1)\mathrm{d}t$$

(b)

$$\int_0^\infty 2^{-x}\mathrm{d}x$$

(c)

$$\int \frac{5}{\sqrt{9 - 25x^2}}\mathrm{d}x$$

Q4 Evaluate the following integrals.

(a)

$$\int \frac{x^2}{(x^2+1)^{5/2}} \mathrm{d}x$$

(b)

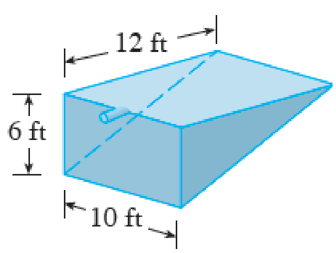
$$\int_8^\infty \frac{10}{x^2-4x-21} \mathrm{d}x$$

Q5 The solid is generated by revolving the curve $y = \cos x - 1$ for $0 \le x \le \pi/2$ about the axis $y = 1$.

(a) Sketch the solid and set up an integral for the volume of it.

(b) Find the volume of the rotating solid.

Q6 A tank (shown below) is full of oil weighing 10 lb/ft^3. Find the work required to pump the oil out of the spout. The base is a $10 \times 6\sqrt{3}$ rectangle. The back end is a 6×10 rectangle, the two sides are right triangle with height 6, base $6\sqrt{3}$ and hypotenuse 12 (all in ft). The spout is 6 ft from the base.



Integrals

- **Volume**: Suppose $A(x)$ is the cross-sectional area of the solid S perpendicular to the x-axis, then the volume of S is given by

$$V = \int_a^b A(x)\ dx$$

- **Work**: Suppose $f(x)$ is a force function. The work in moving an object from a to b is given by:

$$W = \int_a^b f(x)\ dx$$

- $\displaystyle\int \frac{1}{x}\ dx = \ln|x| + C$
- $\displaystyle\int \tan x\ dx = \ln|\sec x| + C$
- $\displaystyle\int \sec x\ dx = \ln|\sec x + \tan x| + C$
- $\displaystyle\int a^x\ dx = \frac{a^x}{\ln a} + C \qquad \text{for } a \neq 1$
- **Integration by Parts**:

$$\int u\ dv = uv - \int v\ du$$

Derivatives

- $\displaystyle\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$
- Inverse Trigonometric Functions:

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

- If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at a and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Hyperbolic and Trig Identities

- Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch}(x) = \frac{1}{\sinh x}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech}(x) = \frac{1}{\cosh x}$$

$$\tanh(x) = \frac{\sinh x}{\cosh x} \qquad \coth(x) = \frac{\cosh x}{\sinh x}$$

- $\cosh^2 x - \sinh^2 x = 1$
- $\cos^2 x + \sin^2 x = 1$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(2x) = 2\sin x \cos x$
- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$